

CLIFTON HIGH SCHOOL MATHEMATICS SUMMER WORKBOOK

CALCULUS HONORS 12th Grade

Completion of this summer work is required on the first day of the school year.

Date Received: _____	Date Completed: _____
Student Signature: _____	
Parent Signature: _____	

Formula Sheet

Trig Identities

$$\sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x} \quad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Equations of lines

Slope-Intercept form $y = mx + b$

Point-Slope form $y - y_1 = m(x - x_1)$

Normal line is perpendicular to tangent line

Radicals

If $x^2 = a$, then $x = \pm\sqrt{a}$

Sum and difference of cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Exponents

$$a^0 = 1, a \neq 0$$

$$a^1 = a$$

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}, a \neq 0$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

Logarithms

$$\ln 1 = 0$$

$$\ln e = 1$$

$$\ln mn = \ln m + \ln n$$

$$\ln \frac{m}{n} = \ln m - \ln n$$

$$\ln m^n = n \ln m$$

$$e^{\ln x} = x = \ln e^x$$

$$\log_b x = \frac{\ln x}{\ln a}$$

Conversion formula:

$$\log_b x = y$$

$$\Leftrightarrow$$

$$b^y = x$$

*In Calculus, it's rarely the calculus that'll get you; it's the algebra. Students entering Calculus must have a strong foundation in algebra. Know how to factor, simplify, expand, manipulate fractions, and solve equations.

Complete the following questions on a separate piece of paper.
Show all work for full credit.

Name _____

Expand using properties of logarithms.

1. $\ln x^2y^3$

2. $\ln \frac{x+3}{4y}$

3. $\ln 3\sqrt{x}$

4. $\ln 4xy$

Condense into the logarithm of a single expression

5. $4 \ln x + 5 \ln y$

6. $\frac{2}{3} \ln a + 5 \ln 2$

7. $\ln x - \ln 2$

8. $2 \ln 3 - \frac{1}{2} \ln (x^2 + 1)$

Solve. Give your answer in exact form and rounded to three decimal places.

9. $\ln (x + 3) = 2$

10. $\ln x + \ln 4 = 1$

11. $\ln (x + 1) - \ln (2x - 3) = \ln 2$

12. $e^{4x+5} = 1$

13. $2^x = 8^{4x-1}$

14. $100e^{x/\ln 4} = 50$

15. Solve for x (factor first)
 $(x + 2)^2(x + 6)^3 + (x + 2)(x + 6)^4 = 0$

16. Solve for x
 $2\sqrt{x+3} = x + 3$

17. Find the product of:

$$\frac{4x^2 - 1}{2x^2 - 5x - 3} \text{ and } \frac{x^2 - 6x + 9}{2x^2 + 5x - 3}$$

18. Solve for a (factor first)
 $a(3a + 2)^{\frac{1}{2}} + 2(3a + 2)^{\frac{3}{2}} = 0$

19. Divide:
 $\frac{y^2 - 25}{(y + 5)^2} \div \frac{2y - 10}{4y + 20}$

20. Simplify:

$$\frac{3}{x - 4} \div \left(1 - \frac{2}{x - 4}\right)$$

21. Add:

$$\frac{1}{x^2 - 2x - 3} + \frac{1}{x^2 - 4x + 3}$$

22. Simplify:

$$\frac{\frac{a}{2} - \frac{8}{a}}{\frac{1}{4} - \frac{1}{a}}$$

23. Subtract:

$$\frac{5x}{2y + 4} - \frac{6}{y^2 + 2y}$$

Factor the following:

24. $121x^2 - 225y^4$

25. $x^2 - 8x + 16$

26. $3x^2 + 13x - 10$

27. $6x^2 - 11x + 4$

28. $2x^2 - 16x + 30$

29. $4x^2 + 8x + 3$

30. $16x^2 + 56xy + 49y^2$

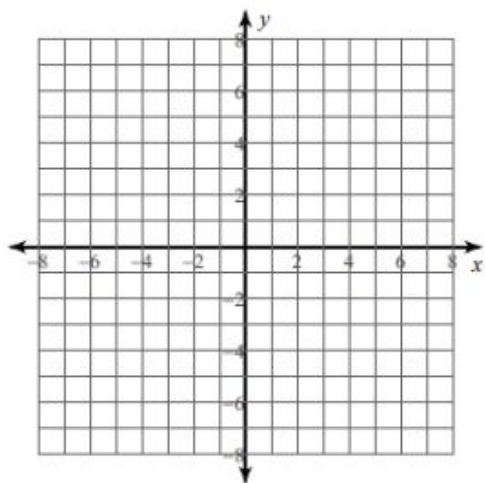
31. $8x^4 + 44x^3 + 56x^2$

32. $6x^3y^2 + 54x^2y^2 - 312xy^2$

Sketch the graph of each piecewise defined function:

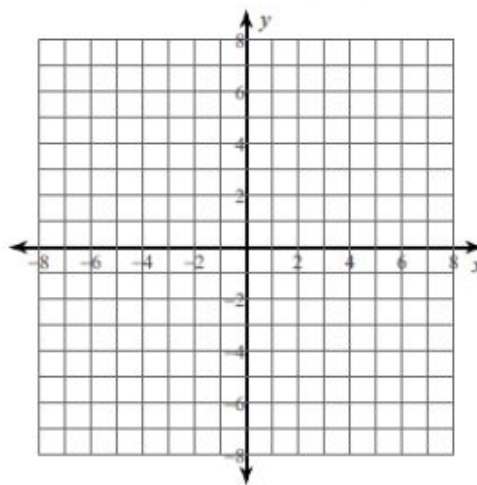
33.

$$f(x) = \begin{cases} -2x - 1, & x \leq 2 \\ -x + 4, & x > 2 \end{cases}$$



34.

$$f(x) = \begin{cases} -4, & x \leq -2 \\ x - 2, & -2 < x < 2 \\ -2x + 4, & x \geq 2 \end{cases}$$



Without a calculator, fill out the following tables. Show all work.

35.

$\sin \pi =$	$\csc \pi =$
$\cos \pi =$	$\sec \pi =$
$\tan \pi =$	$\cot \pi =$

36.

$\sin \pi/2 =$	$\csc \pi/2 =$
$\cos \pi/2 =$	$\sec \pi/2 =$
$\tan \pi/2 =$	$\cot \pi/2 =$

Write your answer in slope-intercept form.

37. Write the equation of the line through the points $(-1, 5)$ and $(2, 7)$.
38. Write the equation of the line through the points $(-5, 6)$ and $(3, 6)$.
39. Write the equation of the line through the points $(-4, 1)$ and $(-4, 16)$.
40. Write the equation of the line parallel to $y = -3x + 1$ through the point $(2, 6)$.
41. Write the equation of the line perpendicular to $y = 5x - 3$ through the point $(-1, 2)$.
42. Write the equation of the line perpendicular to $3x - 2y = 8$ through the point $(0, 4)$.
43. Write the equation of the line perpendicular to $y = 4$ through the point $(2, -5)$.
44. Write the equation of the line perpendicular to $x = -10$ through the point $(6, 7)$.

Simplify using properties of exponents.

45. $(2x^0y^2)^{-3}2yx^3$

46. $\frac{2y^33xy^3}{3x^2y^4}$

47. $(n^3)^32n^{-1}$